

DIFFERENTIATION / INTEGRATION : RATIONAL POWERS OF X EXERCISE

Exercise A

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

- 1. $y = 6x^{\frac{2}{3}}$
- 2. $y = x^2 + \frac{1}{x^2}$
- 3. $y = 4\sqrt{x}$
- 4. $y = \frac{3}{x} - \frac{4}{x^3}$
- 5. $y = x^{\frac{7}{4}}$

Evaluate $\frac{dy}{dx}$ when $x = 4$

- 6. $y = 3x + \sqrt{x}$
- 7. $y = x^{-\frac{1}{2}}$
- 8. $y = x^2 - \frac{1}{x^2}$
- 9. $y = \frac{x^{5/2}}{5}$
- 10. $y = \frac{1}{x} - x^{\frac{1}{2}}$

Exercise B

Find the following indefinite integrals:

- 1. $\int 3\sqrt{x} \, dx$
- 2. $\int \left(\frac{1}{x^2} - x\right) \, dx$
- 3. $\int x^{-\frac{1}{3}} \, dx$
- 4. $\int \left(\frac{1}{x^4} - \frac{1}{x^3}\right) \, dx$
- 5. $\int \left(x^3 + \frac{1}{x^3}\right) \, dx$

Evaluate the following definite integrals

- 6. $\int_1^2 \frac{1}{x^2} \, dx$
- 7. $\int_2^3 \frac{1}{x^3} \, dx$
- 8. $\int_4^9 \frac{1}{\sqrt{x}} \, dx$
- 9. $\int_1^2 \left(x - \frac{1}{x^4}\right) \, dx$
- 10. $\int_0^1 \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}}\right) \, dx$

Exercise A

$$1. \quad y = 6x^{2/3}$$

$$\frac{dy}{dx} = \frac{2}{3} \times 6x^{-1/3}$$

$$\frac{dy}{dx} = 4x^{-1/3}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{3} \times 4x^{-4/3}$$

$$\frac{d^2y}{dx^2} = -\frac{4}{3}x^{-4/3}$$

$$2. \quad y = x^2 + \frac{1}{x^2}$$

$$y = x^2 + x^{-2}$$

$$\frac{dy}{dx} = 2x - 2x^{-3}$$

$$\text{or } \frac{dy}{dx} = 2x - \frac{2}{x^3}$$

$$\frac{d^2y}{dx^2} = 2 - 2(-3)x^{-4}$$

$$\frac{d^2y}{dx^2} = 2 + 6x^{-4}$$

$$\text{or } \frac{d^2y}{dx^2} = 2 + \frac{6}{x^4}$$

$$3. \quad y = 4\sqrt{x}$$

$$y = 4x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2} \times 4x^{-1/2}$$

$$\frac{dy}{dx} = 2x^{-1/2}$$

$$\text{or } \frac{dy}{dx} = \frac{2}{\sqrt{x}}$$

$$\frac{d^2y}{dx^2} = -\frac{1}{2} \times 2x^{-3/2}$$

$$\frac{d^2y}{dx^2} = -x^{-3/2}$$

Note

Where the solutions say 'or', it is not necessary to rewrite the answer in the form given in the question. As long as the powers of x are equivalent, say $x^{1/2} = \sqrt{x}$, full marks will be awarded.

4.

$$y = \frac{3}{x} - \frac{4}{x^3}$$

$$y = 3x^{-1} - 4x^{-3}$$

$$\frac{dy}{dx} = -3x^{-2} + 12x^{-4}$$

$$\text{or } \frac{dy}{dx} = -\frac{3}{x^2} + \frac{12}{x^4}$$

When $x = 4$

$$\frac{dy}{dx} = 3 + \frac{1}{2\sqrt{4}}$$

$$\frac{dy}{dx} = 3 + \frac{1}{4}$$

$$\frac{dy}{dx} = 3\frac{1}{4} \text{ or } \frac{13}{4}$$

$$\frac{d^2y}{dx^2} = +6x^{-3} - 48x^{-5}$$

$$\text{or } \frac{d^2y}{dx^2} = \frac{6}{x^3} - \frac{48}{x^5}$$

7. $y = x^{-\frac{1}{2}}$

$$\frac{dy}{dx} = -\frac{1}{2}x^{-\frac{3}{2}}$$

$$\text{or } \frac{dy}{dx} = -\frac{1}{2x^{\frac{3}{2}}}$$

When $x = 4$

$$\frac{dy}{dx} = -\frac{1}{2(4)^{\frac{3}{2}}}$$

$$\frac{dy}{dx} = -\frac{1}{2 \times 8}$$

$$\frac{dy}{dx} = -\frac{1}{16}$$

5.

$$y = x^{\frac{7}{4}}$$

$$\frac{dy}{dx} = \frac{7}{4}x^{\frac{3}{4}}$$

$$\frac{d^2y}{dx^2} = \frac{3}{4} \times \frac{7}{4}x^{-\frac{1}{4}}$$

$$\frac{d^2y}{dx^2} = \frac{21}{16}x^{-\frac{1}{4}}$$

6.

$$y = 3x + \sqrt{x}$$

$$y = 3x + x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 3 + \frac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = 3 + \frac{1}{2\sqrt{x}}$$

8.

$$y = x^2 - \frac{1}{x^2}$$

$$y = x^2 - x^{-2}$$

$$\frac{dy}{dx} = 2x + 2x^{-3}$$

$$\frac{dy}{dx} = 2x + \frac{2}{x^3}$$

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8.
contWhen $x = 4$

$$\frac{dy}{dx} = 2(4) + \frac{2}{4^3}$$

$$\frac{dy}{dx} = 8 + \frac{2}{64}$$

$$\frac{dy}{dx} = 8\frac{1}{32}$$

$$\frac{dy}{dx} = -\frac{1}{x^2} - \frac{1}{2\sqrt{x}}$$

When $x = 4$

$$\frac{dy}{dx} = -\frac{1}{4^2} - \frac{1}{2\sqrt{4}}$$

$$\frac{dy}{dx} = -\frac{1}{16} - \frac{1}{4}$$

$$\frac{dy}{dx} = -\frac{5}{16}$$

||

9.

$$y = \frac{x^{5/2}}{5}$$

$$\frac{dy}{dx} = \frac{5}{2} \times \frac{x^{3/2}}{5}$$

$$\frac{dy}{dx} = \frac{1}{2} x^{3/2}$$

When $x = 4$

$$\frac{dy}{dx} = \frac{1}{2} (4)^{3/2}$$

$$\frac{dy}{dx} = \frac{1}{2} \times 8$$

$$\frac{dy}{dx} = 4$$

10.

$$y = \frac{1}{x} - x^{\frac{1}{2}}$$

$$y = x^{-1} - x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = -1x^{-2} - \frac{1}{2}x^{-\frac{1}{2}}$$

Exercise B

1.

$$\begin{aligned} & \int 3\sqrt{x} \, dx \\ &= \int 3x^{\frac{1}{2}} \, dx \\ &= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + C \\ &= \frac{2}{3} \times 3x^{\frac{3}{2}} + C \\ &= 2x^{\frac{3}{2}} + C \end{aligned}$$

2.

$$\begin{aligned} & \int \left(\frac{1}{x^2} - x \right) dx \\ &= \int (x^{-2} - x) dx \\ &= \frac{x^{-1}}{-1} - \frac{x^2}{2} + C \\ &= -\frac{1}{x} - \frac{x^2}{2} + C \end{aligned}$$

3.

$$\begin{aligned} & \int x^{-\frac{1}{3}} dx \\ &= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} + C \\ &= \frac{3}{2} x^{\frac{2}{3}} + C \end{aligned}$$

$$\begin{aligned} 4. \quad & \int \left(\frac{1}{x^4} - \frac{1}{x^3} \right) dx \\ &= \int (x^{-4} - x^{-3}) dx \\ &= \frac{x^{-3}}{-3} - \frac{x^{-2}}{-2} + C \\ &= -\frac{1}{3x^3} + \frac{1}{2x^2} + C \end{aligned}$$

$$\begin{aligned} 5. \quad & \int \left(x^3 + \frac{1}{x^3} \right) dx \\ &= \int (x^3 + x^{-3}) dx \\ &= \frac{x^4}{4} + \frac{x^{-2}}{-2} + C \\ &= \frac{x^4}{4} - \frac{1}{2x^2} + C \end{aligned}$$

$$\begin{aligned} 6. \quad & \int_1^2 \frac{1}{x^2} dx \\ &= \int_1^2 x^{-2} dx \\ &= \left[\frac{x^{-1}}{-1} \right]_1^2 \\ &= \left[-\frac{1}{x} \right]_1^2 \\ &= \left(-\frac{1}{2} \right) - \left(-\frac{1}{1} \right) \end{aligned}$$

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6 cont.

$$= -\frac{1}{2} + 1$$

$$= \frac{1}{2}$$

$$= \left[\frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_4^9$$

$$= \left[2\sqrt{x} \right]_4^9$$

$$= (2\sqrt{9}) - (2\sqrt{4})$$

$$= 6 - 4$$

$$= 2$$

7.

$$\int_2^3 \frac{1}{x^3} dx$$

$$= \int_2^3 x^{-3} dx$$

$$= \left[\frac{x^{-2}}{-2} \right]_2^3$$

$$= \left[-\frac{1}{2x^2} \right]_2^3$$

$$= \left(-\frac{1}{2 \times 3^2} \right) - \left(-\frac{1}{2 \times 2^2} \right)$$

$$= -\frac{1}{18} + \frac{1}{8}$$

$$= -\frac{4}{72} + \frac{9}{72}$$

$$= \frac{5}{72}$$

9. $\int_1^2 \left(x - \frac{1}{x^4} \right) dx$

$$= \int_1^2 \left(x - x^{-4} \right) dx$$

$$= \left[\frac{x^2}{2} - \frac{x^{-3}}{-3} \right]_1^2$$

$$= \left[\frac{x^2}{2} + \frac{1}{3x^3} \right]_1^2$$

$$= \left(\frac{2^2}{2} + \frac{1}{3(2)^3} \right) - \left(\frac{1^2}{2} + \frac{1}{3(1)^3} \right)$$

$$= 2 + \frac{1}{24} - \frac{1}{2} - \frac{1}{3}$$

$$= \frac{48}{24} + \frac{1}{24} - \frac{12}{24} - \frac{8}{24}$$

$$= \frac{29}{24} \quad \text{or} \quad \frac{5}{24}$$

8.

$$\int_4^9 \frac{1}{\sqrt{x}} dx$$

$$= \int_4^9 x^{-\frac{1}{2}} dx$$

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10.
$$\int_0^1 \left(x^{\frac{1}{3}} + x^{-\frac{1}{3}} \right) dx$$

$$= \left[\frac{x^{\frac{4}{3}}}{\frac{4}{3}} + \frac{x^{\frac{2}{3}}}{\frac{2}{3}} \right]_0^1$$
$$= \left[\frac{3}{4} x^{\frac{4}{3}} + \frac{3}{2} x^{\frac{2}{3}} \right]_0^1$$
$$= \left(\frac{3}{4} (1)^{\frac{4}{3}} + \frac{3}{2} (1)^{\frac{2}{3}} \right) - (0 + 0)$$
$$= \frac{3}{4} + \frac{3}{2}$$
$$= \frac{3}{4} + \frac{6}{4}$$
$$= \frac{9}{4} \quad \text{or} \quad 2\frac{1}{4}$$

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