DIFFERENTIATION/INTEGRATION: RATIONAL POWERS OF x — TRANSCRIPT

Standard Formulae:

Differentiation

$$\frac{d}{dx}\, a x^{n} = n a x^{n-1}$$

Integration

$$\int a x^{n}\, dx = \frac{a x^{n+1}}{n+1} + c$$

for $n \neq -1$

Differentiation: Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

Example 1

$$y = 3\sqrt{x} - \frac{5}{x}$$

$$y = 3x^{\frac{1}{2}} - 5x^{-1}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + 5x^{-2}$$

$$\frac{dy}{dx} = \frac{3}{2\sqrt{x}} + \frac{5}{x^2}$$

(This last line is optional)

$$\frac{dy}{dx} = \frac{3}{2}x^{-\frac{1}{2}} + 5x^{-2}$$

$$\frac{d^2y}{dx^2} = -\frac{3}{4}x^{-3/2} - 10x^{-3}$$

Example 2

$$y = x^{\frac{1}{4}} + \frac{1}{x^3}$$

$$y = x^{\frac{1}{4}} + x^{-3}$$

$$\frac{dy}{dx} = \frac{1}{4}x^{-\frac{3}{4}} - 3x^{-4}$$

$$\frac{d^2y}{dx^2} = -\frac{3}{16}x^{-7/4} + 12x^{-5}$$

Example 3

Find $\frac{dy}{dx}$ when $x = 16$

$$y = \sqrt{x} + \frac{1}{x^2}$$

$$y = x^{\frac{1}{2}} + x^{-2}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 2x^{-3}$$

$$= \frac{1}{2\sqrt{x}} - \frac{2}{x^3}$$

When $x = 16$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{16}} - \frac{2}{16^3}$$

$$= \frac{1}{8} - \frac{2}{4096} = \frac{255}{2048}$$

Example 4

Find $\frac{dy}{dx}$ when $x = 9$

$$y = x^{3/2} - x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}}$$

When $x = 9$

$$\frac{dy}{dx} = \frac{3}{2}\sqrt{9} - \frac{1}{2\sqrt{9}}$$

$$= \frac{9}{2} - \frac{1}{6}$$

$$= \frac{27}{6} - \frac{1}{6}$$

$$= \frac{26}{6}$$

$$= 4\frac{1}{3}$$

Integration:

Evaluate the indefinite integrals.

Example 5

$$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}} \right) dx$$

$$= \int \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}} \right) dx$$

$$= \frac{x^{3/2}}{\frac{3}{2}} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{2}{3} x^{3/2} + 2x^{1/2} + c$$

Example 6

$$\int \left(\frac{3}{x^2} - \frac{2}{x^3} \right) dx$$

$$= \int \left(3x^{-2} - 2x^{-3} \right) dx$$

$$= \frac{3x^{-1}}{-1} - \frac{2x^{-2}}{-2} + c$$

$$= -3x^{-1} + x^{-2} + c$$

$$= -\frac{3}{x} + \frac{1}{x^2} + c$$

Evaluate the definite integrals.

Example 7

$$\int_1^8 \left(x^{-2/3} + \frac{1}{x^2} \right) dx$$

$$= \int_1^8 \left(x^{-2/3} + x^{-2} \right) dx$$

$$= \left[\frac{x^{1/3}}{\frac{1}{3}} + \frac{x^{-1}}{-1} \right]_1^8$$

$$= \left[3x^{\frac{1}{3}} - \frac{1}{x} \right]_1^8$$

$$= \left(3 \times 8^{\frac{1}{3}} - \frac{1}{8} \right) - \left(3 \times 1^{\frac{1}{3}} - \frac{1}{1} \right)$$

$$= \left(6 - \frac{1}{8} \right) - \left(3 - 1 \right)$$

$$= 6 - \frac{1}{8} - 2 = 3\frac{7}{8}$$

Example 8

$$\int_1^4 \left(\frac{1}{x^3} - \sqrt{x} \right) dx$$

$$= \int_1^4 \left(x^{-3} - x^{\frac{1}{2}} \right) dx$$

$$= \left[\frac{x^{-2}}{-2} - \frac{x^{3/2}}{3/2} \right]_1^4$$

$$= \left[-\frac{1}{2x^2} - \frac{2}{3} x^{3/2} \right]_1^4$$

$$= \left(-\frac{1}{2(4)^2} - \frac{2}{3}(4)^{3/2} \right) - \left(-\frac{1}{2} - \frac{2}{3} \right)$$

$$= \left(-\frac{1}{32} - \frac{16}{3} \right) - \left(-\frac{1}{2} - \frac{2}{3} \right)$$

$$= -\frac{1}{32} - \frac{16}{3} + \frac{1}{2} + \frac{2}{3}$$

$$= \frac{15}{32} - \frac{14}{3}$$

$$= -4 \tfrac{19}{96}$$