QUADRATIC EQUATIONS BY FORMULA

To solve the general quadratic equation

$$a x^2 + bx + c = 0$$

the formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1

Solve $x^2 - 5x + 6 = 0$

$a = 1, \quad b = -5, \quad c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+5 \pm \sqrt{(-5)^2 - 4 \times 1 \times 6}}{2 \times 1}$$

$$x = \frac{+5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{+5 \pm 1}{2}$$

$$x = \frac{6}{2} \qquad or \qquad x = \frac{4}{2}$$

$$x = 3 \qquad or \qquad x = 2$$

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Example 2

Solve $x^2 + 2x - 15 = 0$

$a = 1, \quad b = 2, \quad c = -15$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4 \times 1 \times (-15)}}{2 \times 1}$$

$$x = \frac{-2 \pm \sqrt{4 + 60}}{2}$$

$$x = \frac{-2 \pm 8}{2}$$

$$x = \frac{6}{2} \qquad \text{or} \qquad x = \frac{-10}{2}$$

$$x = 3 \qquad \text{or} \qquad x = -5$$

Both these examples of quadratic equations could have been solved by factorising.

$x^2 - 5x + 6 = 0$

$(x - 3)(x - 2) = 0$

$\Rightarrow x - 3 = 0 \quad \text{or} \quad x - 2 = 0$

$\quad x = 3 \qquad\qquad x = 2$

$x^2 + 2x - 15 = 0$

$(x + 5)(x - 3) = 0$

$\Rightarrow x + 5 = 0 \quad \text{or} \quad x - 3 = 0$

$\quad x = -5 \qquad\qquad x = 3$

QUADRATIC EQUATIONS BY FORMULA

The quadratic formula is particularly useful when quadratic equations do not factorise. If a question asks for answers to 3 significant figures, that is a good indication that the equation will not factorise.

Example 3

Solve $3x^2 - 5x - 7 = 0$

giving answers to 3 sig. fig.

$a = 3, \quad b = -5, \quad c = -7$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{+5 \pm \sqrt{(-5)^2 - 4 \times 3 \times (-7)}}{2 \times 3}$$

$$x = \frac{+5 \pm \sqrt{25 + 84}}{6}$$

$$x = \frac{+5 \pm \sqrt{109}}{6}$$

$$x = \frac{+5 + \sqrt{109}}{6} \qquad or \qquad x = \frac{+5 - \sqrt{109}}{6}$$

$$x = 2.57 \qquad or \qquad x = -0.907$$

Example 4

Solve $5x^2 + 11x + 3 = 0$

giving answers to 3 sig. fig.

$a = 5, \quad b = 11, \quad c = 3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-11 \pm \sqrt{11^2 - 4 \times 5 \times 3}}{2 \times 5}$$

$$x = \frac{-11 \pm \sqrt{121 - 60}}{10}$$

$$x = \frac{-11 \pm \sqrt{61}}{10}$$

$$x = \frac{-11 + \sqrt{61}}{10} \quad \text{or} \quad x = \frac{-11 - \sqrt{61}}{10}$$

$$x = -0.319 \quad \text{or} \quad x = -1.88$$

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