INTEGRATION BY SUBSTITUTION

Ex 1 $\int (2x+1)(x-3)^4 \, dx$

Let $u = x-3$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Also $x = u+3$

$= \int (2(u+3)+1) u^4 \, du$

$= \int (2u+7) u^4 \, du$

$= \int (2u^5 + 7u^4) \, du$

$= \frac{2u^6}{6} + \frac{7u^5}{5} + c$

$= \frac{1}{3}(x-3)^6 + \frac{7}{5}(x-3)^5 + c$

INTEGRATION BY SUBSTITUTION

Ex 2

$$\int \left(\frac{x}{x-2}\right) dx$$

Let $u = x - 2$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Also $x = u + 2$

$$= \int \left(\frac{u+2}{u}\right) du$$

$$= \int \left(\frac{u}{u} + \frac{2}{u}\right) du$$

$$= \int \left(1 + \frac{2}{u}\right) du$$

$$= u + 2\ln u + c$$

$$= x - 2 + 2\ln|x-2| + c$$

$$= x + 2\ln|x-2| + c \qquad \text{(since } c \text{ can incorporate } -2\text{)}$$

INTEGRATION BY SUBSTITUTION

Ex 3

$$\int x\sqrt{3x-1}\,dx$$

Let $u = 3x - 1$

$\Rightarrow \frac{du}{dx} = 3$

$\Rightarrow du = 3\,dx$

$\Rightarrow \frac{1}{3}du = dx$

Also $3x = u + 1$

$x = \frac{u+1}{3}$

$$= \int \left(\frac{u+1}{3}\right) u^{\frac{1}{2}} \cdot \frac{1}{3}\,du$$

$$= \int \frac{1}{9}\left(u^{3/2} + u^{1/2}\right) du$$

$$= \frac{1}{9}\left(\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2}\right) + c$$

$$= \frac{1}{9}\left(\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right) + c$$

$$= \frac{2}{45}(3x-1)^{5/2} + \frac{2}{27}(3x-1)^{3/2} + c$$

INTEGRATION BY SUBSTITUTION

Ex 4

$$\int x\sqrt{x^2-1}\,dx$$

Let $u = x^2 - 1$

$\Rightarrow \frac{du}{dx} = 2x$

$\Rightarrow du = 2x\,dx$

$\Rightarrow \frac{1}{2}du = x\,dx$

$$= \int \sqrt{u}\,\frac{1}{2}\,du$$

$$= \int \frac{1}{2}u^{\frac{1}{2}}\,du$$

$$= \frac{1}{2}\cdot\frac{u^{3/2}}{\frac{3}{2}} + c$$

$$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2} + c$$

$$= \frac{1}{3}\left(x^2-1\right)^{3/2} + c$$

INTEGRATION BY SUBSTITUTION

Ex 5

$$\int 6\sin x \cos^3 x \, dx$$

Let $u = \cos x$

$\Rightarrow \frac{du}{dx} = -\sin x$

$\Rightarrow du = -\sin x \, dx$

$\Rightarrow -du = \sin x \, dx$

$$= \int 6u^3(-du)$$

$$= -\int 6u^3 \, du$$

$$= -\frac{6u^4}{4} + c$$

$$= -\frac{3}{2}\cos^4 x + c$$

DEFINITE INTEGRATION BY SUBSTITUTION

Ex 6 $\int_0^1 \frac{1}{\sqrt{x+2}}\,dx$

Let $u = x+2$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Change limits

When $x = 1$, $u = 3$

when $x = 0$, $u = 2$

$= \int_2^3 \frac{1}{\sqrt{u}}\,du$

$= \int_2^3 u^{-\frac{1}{2}}\,du$

$= \left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right]_2^3$

$= \left[2u^{\frac{1}{2}} \right]_2^3$

$= 2\sqrt{3} - 2\sqrt{2}$

$= 2\left(\sqrt{3} - \sqrt{2}\right)$

DEFINITE INTEGRATION BY SUBSTITUTION

Ex 7 $\int_0^2 3x e^{2x^2} dx$

Let $u = 2x^2$

$\Rightarrow \frac{du}{dx} = 4x$

$\Rightarrow du = 4x\,dx$

$\Rightarrow \frac{1}{4} du = x\,dx$

Change limits

when $x = 2$, $u = 2(2)^2 = 8$

when $x = 0$, $u = 2(0)^2 = 0$

$= \int_0^8 \frac{3}{4} e^u du$

$= \frac{3}{4} \left[e^u \right]_0^8$

$= \frac{3}{4} \left(e^8 - e^0 \right)$

$= \frac{3}{4} \left(e^8 - 1 \right)$