

INTEGRATION BY SUBSTITUTION

Ex 1

$$\int (2x+1)(x-3)^4 dx$$

Let  $u = x-3$

$$\Rightarrow \frac{du}{dx} = 1$$

$$\Rightarrow du = dx$$

Also  $x = u+3$

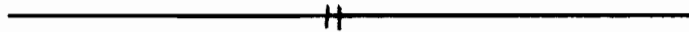
$$= \int (2(u+3)+1) u^4 du$$

$$= \int (2u+7) u^4 du$$

$$= \int (2u^5 + 7u^4) du$$

$$= \frac{2u^6}{6} + \frac{7u^5}{5} + c$$

$$= \frac{1}{3}(x-3)^6 + \frac{7}{5}(x-3)^5 + c$$



## INTEGRATION BY SUBSTITUTION

Ex 2

$$\int \left( \frac{x}{x-2} \right) dx$$

$$\text{Let } u = x - 2$$

$$\Rightarrow \frac{du}{dx} = 1$$

$$\Rightarrow du = dx$$

$$\text{Also } x = u + 2$$

$$= \int \left( \frac{u+2}{u} \right) du$$

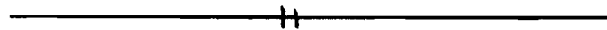
$$= \int \left( \frac{u}{u} + \frac{2}{u} \right) du$$

$$= \int \left( 1 + \frac{2}{u} \right) du$$

$$= u + 2 \ln u + C$$

$$= x - 2 + 2 \ln |x - 2| + C$$

$$= x + 2 \ln |x - 2| + C \quad (\text{since } C \text{ can incorporate } -2)$$



INTEGRATION BY SUBSTITUTION

Ex 3

$$\int x \sqrt{3x-1} \, dx$$

Let  $u = 3x - 1$

$$\Rightarrow \frac{du}{dx} = 3$$

$$\Rightarrow du = 3dx$$

$$\Rightarrow \frac{1}{3} du = dx$$

Also  $3x = u + 1$

$$x = \frac{u+1}{3}$$

$$= \int \left( \frac{u+1}{3} \right) u^{\frac{1}{2}} \cdot \frac{1}{3} du$$

$$= \int \frac{1}{9} (u^{3/2} + u^{1/2}) du$$

$$= \frac{1}{9} \left( \frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} \right) + C$$

$$= \frac{1}{9} \left( \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} \right) + C$$

$$= \frac{2}{45} (3x-1)^{5/2} + \frac{2}{27} (3x-1)^{3/2} + C$$

INTEGRATION BY SUBSTITUTION

Ex 4

$$\int x \sqrt{x^2 - 1} dx$$

$$\text{Let } u = x^2 - 1$$

$$\Rightarrow \frac{du}{dx} = 2x$$

$$\Rightarrow du = 2x dx$$

$$\Rightarrow \frac{1}{2} du = x dx$$

$$= \int \sqrt{u} \frac{1}{2} du$$

$$= \int \frac{1}{2} u^{\frac{1}{2}} du$$

$$= \frac{1}{2} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{1}{2} \cdot \frac{2}{3} u^{\frac{3}{2}} + C$$

$$= \frac{1}{3} (x^2 - 1)^{\frac{3}{2}} + C$$

## INTEGRATION BY SUBSTITUTION

Ex 5

$$\int 6 \sin x \cos^3 x \, dx$$

$$\text{Let } u = \cos x$$

$$\Rightarrow \frac{du}{dx} = -\sin x$$

$$\Rightarrow du = -\sin x \, dx$$

$$\Rightarrow -du = \sin x \, dx$$

$$= \int 6u^3(-du)$$

$$= - \int 6u^3 \, du$$

$$= - \frac{6u^4}{4} + C$$

$$= - \frac{3}{2} \cos^4 x + C$$

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DEFINITE INTEGRATION BY SUBSTITUTION

Ex 6

$$\int_0^1 \frac{1}{\sqrt{x+2}} dx$$

Let  $u = x+2$

$$\Rightarrow \frac{du}{dx} = 1$$

$$\Rightarrow du = dx$$

Change limits

when  $x = 1$ ,  $u = 3$

when  $x = 0$ ,  $u = 2$

$$= \int_2^3 \frac{1}{\sqrt{u}} du$$

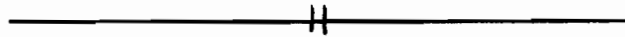
$$= \int_2^3 u^{-\frac{1}{2}} du$$

$$= \left[ \frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right]_2^3$$

$$= \left[ 2u^{\frac{1}{2}} \right]_2^3$$

$$= 2\sqrt{3} - 2\sqrt{2}$$

$$= 2(\sqrt{3} - \sqrt{2})$$



DEFINITE INTEGRATION BY SUBSTITUTION

Ex 7

$$\int_0^2 3x e^{2x^2} dx$$

$$\text{Let } u = 2x^2$$

$$\Rightarrow \frac{du}{dx} = 4x$$

$$\Rightarrow du = 4x dx$$

$$\Rightarrow \frac{1}{4} du = x dx$$

Change limits

$$\text{when } x = 2, u = 2(2)^2 = 8$$

$$\text{when } x = 0, u = 2(0)^2 = 0$$

$$= \int_0^8 \frac{3}{4} e^u du$$

$$= \frac{3}{4} \left[ e^u \right]_0^8$$

$$= \frac{3}{4} (e^8 - e^0)$$

$$= \frac{3}{4} (e^8 - 1)$$