SOLVING QUADRATIC EQUATIONS BY FACTORISING (2) TRANSCRIPT

Consider

$$(ax+c)(bx+d)$$

$$= abx^2 + bcx + adx + cd$$

$$= abx^2 + (bc+ad)x + cd$$

This is obviously a more complex relationship than when there was only a single x^2.

However, it is still possible to work back from the expansion to the brackets

Begin by multiplying the number of x^2 by the constant term. This gives

$$ab \times cd$$

$$= abcd$$

Now identify a pair of factors of abcd that add up to the number of x

In effect this process identifies $bc + ad$

Now we split the number of x into two parts to give

$$abx^2 + bcx + adx + cd$$

Now factorise the front two terms and factorise the back two terms to give

$$bx(ax+c) + d(ax+c)$$

This leads to a further factorisation

$$(bx+d)(ax+c)$$

as required

We will follow this through by factorising

$$6x^2 + 7x + 2$$

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$6 \times 2 = 12$

$+1 \quad +12$
$-1 \quad -12$
$+2 \quad +6$
$-2 \quad -6$
$+3 \quad +4$ ✓
$-3 \quad -4$

$$6x^2 + 3x + 4x + 2$$
$$= 3x(2x+1) + 2(2x+1)$$
$$= (3x+2)(2x+1)$$

What if we had written the $3x + 4x$ as $4x + 3x$

$$6x^2 + 4x + 3x + 2$$
$$= 2x(3x+2) + 1(3x+2)$$
$$= (2x+1)(3x+2)$$

Same result so it does not matter

Example 1

$$2x^2 + 7x + 3 = 0$$

$2 \times 3 = 6$

$+1 \quad +6$ ✓
$-1 \quad -6$
$+2 \quad +3$
$-2 \quad -3$

$$2x^2 + x + 6x + 3 = 0$$
$$x(2x+1) + 3(2x+1) = 0$$
$$(x+3)(2x+1) = 0$$

Either $x + 3 = 0$

$\Rightarrow x = -3$

or $2x + 1 = 0$

$\Rightarrow 2x = -1$

$\Rightarrow x = -\frac{1}{2}$

Solution
$$\begin{cases} x = -3 \\ x = -\frac{1}{2} \end{cases}$$

We shall now solve 4 equations using this method of factorising:

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Example 2

$$3x^2 + 11x + 10 = 0$$

$3 \times 10 = 30$

$+1$	$+30$
$+2$	$+15$
$+3$	$+10$
$+5$	$+6$ ✓

$$3x^2 + 5x + 6x + 10 = 0$$

$$x(3x+5) + 2(3x+5) = 0$$

$$(x+2)(3x+5) = 0$$

Either $x + 2 = 0$

$\Rightarrow x = -2$

or $3x + 5 = 0$

$\Rightarrow 3x = -5$

$\Rightarrow x = -\frac{5}{3}$

Solution

$$\begin{cases} x = -2 \\ x = -\frac{5}{3} \end{cases}$$

Example 3

$$6x^2 - 7x - 3 = 0$$

$6 \times -3 = -18$

$+1$	-18
-1	$+18$
$+2$	-9 ✓
-2	$+9$
$+3$	-6
-3	$+6$

$$6x^2 + 2x - 9x - 3 = 0$$

$$2x(3x+1) - 3(3x+1) = 0$$

$$(2x-3)(3x+1) = 0$$

Either $2x - 3 = 0$

$\Rightarrow 2x = 3$

$\Rightarrow x = \frac{3}{2}$

or $3x + 1 = 0$

$\Rightarrow 3x = -1$

$\Rightarrow x = -\frac{1}{3}$

Solution

$$\begin{cases} x = 3/2 \\ x = -\frac{1}{3} \end{cases}$$

Example 4

$$4x^2 - 16x + 15 = 0$$

$4 \times 15 = 60$

−1	−60
−2	−30
−3	−20
−4	−15
−5	−12
−6	−10 ✓

$$4x^2 - 6x - 10x + 15 = 0$$

$$2x(2x-3) - 5(2x-3) = 0$$

$$(2x-5)(2x-3) = 0$$

Either $2x - 5 = 0$

$\Rightarrow 2x = 5$

$\Rightarrow x = \frac{5}{2}$

or $2x - 3 = 0$

$\Rightarrow 2x = 3$

$\Rightarrow x = \frac{3}{2}$

Solution $\begin{cases} x = 5/2 \\ x = 3/2 \end{cases}$

Difference of two squares

Consider:

$$(a+b)(a-b)$$

$$= a^2 + ab - ab - b^2$$

$$= a^2 - b^2$$

So $(a+b)(a-b)$ is the factorisation of $a^2 - b^2$

Example $x^2 - 25$

$$= x^2 - 5^2$$

$$= (x+5)(x-5)$$

We will now solve 3 equations using this technique

Example 5

$$x^2 - 16 = 0$$

$$x^2 - 4^2 = 0$$

$$(x+4)(x-4) = 0$$

Either $x + 4 = 0$

$\Rightarrow x = -4$

or $x - 4 = 0$

$\Rightarrow x = 4$

Solution $x = -4, \ x = 4$

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Example 6

$$4x^2 - 49 = 0$$
$$(2x)^2 - 7^2 = 0$$
$$(2x+7)(2x-7) = 0$$

Either $2x + 7 = 0$

$\Rightarrow 2x = -7$

$\Rightarrow x = -\frac{7}{2}$

or $2x - 7 = 0$

$\Rightarrow 2x = 7$

$\Rightarrow x = \frac{7}{2}$

Solution
$$\begin{cases} x = -\frac{7}{2} \\ x = \frac{7}{2} \end{cases}$$

Example 7

$$9x^2 - 1 = 0$$
$$(3x)^2 - 1^2 = 0$$
$$(3x+1)(3x-1) = 0$$

Either $3x + 1 = 0$

$\Rightarrow 3x = -1$

$\Rightarrow x = -\frac{1}{3}$

or $3x - 1 = 0$

$\Rightarrow 3x = 1$

$\Rightarrow x = \frac{1}{3}$

Solution
$$\begin{cases} x = -\frac{1}{3} \\ x = \frac{1}{3} \end{cases}$$