DIFFERENTIATION OF POLYNOMIAL FUNCTIONS (2) — EXERCISE

1. Find the stationary point of the curve: $y = 4x - x^2$

   Determine the nature of the stationary point and sketch the graph.

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2. Find the turning points of the curve: $y = x^3 - 3x^2 - 9x + 1$

   Determine the nature of the turning points and sketch the graph.

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3. Find the turning points of $y = x^2(x-4)^2$

   Determine the nature of the turning points and sketch the graph.

   Hint: Multiply out $x^2(x-4)^2$ first.

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DIFFERENTIATION OF POLYNOMIAL FUNCTIONS (2) EXERCISE

1. $y = 4x - x^2$

$\Rightarrow \dfrac{dy}{dx} = 4 - 2x$

At st. pt. $\dfrac{dy}{dx} = 0$

$\Rightarrow 4 - 2x = 0$

$4 = 2x$

$2 = x$

When $x = 2$

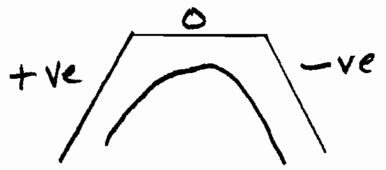
$y = 4(2) - 2^2$

$y = 8 - 4 = 4$

st. pt. at $(2, 4)$

At $x = 1.9$, $\dfrac{dy}{dx} = 4 - 3.8 = 0.2$ +ve

At $x = 2.1$, $\dfrac{dy}{dx} = 4 - 4.2 = -0.2$ -ve



$\therefore (2,4)$ is a max

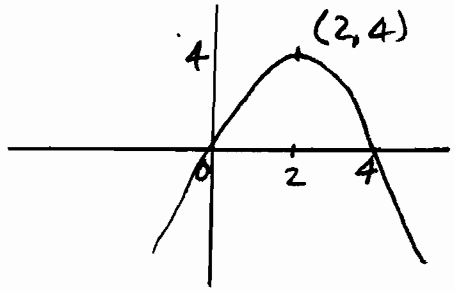
Alternatively,

$\dfrac{d^2y}{dx^2} = -2$

for all values of x

$\therefore$ st. pt. is a maximum

since $\dfrac{d^2y}{dx^2} < 0$



2. $y = x^3 - 3x^2 - 9x + 1$

$\dfrac{dy}{dx} = 3x^2 - 6x - 9$

At st. pt. $\dfrac{dy}{dx} = 0$

$\Rightarrow 3x^2 - 6x - 9 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x - 3)(x + 1) = 0$

$\Rightarrow x = 3$ or $x = -1$

When $x = 3$, $y = 3^3 - 3(3)^2 - 9(3) + 1$

$y = 27 - 27 - 27 + 1$

$y = -26$

St. Point is $(3, -26)$

When $x = -1$, $y = (-1)^3 - 3(-1)^2 - 9(-1) + 1$

$= -1 - 3 + 9 + 1$

$= 6$

St pt at $(-1, 6)$

DIFFERENTIATION OF POLYNOMIAL FUNCTIONS (2) EXERCISE

2 cont)

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

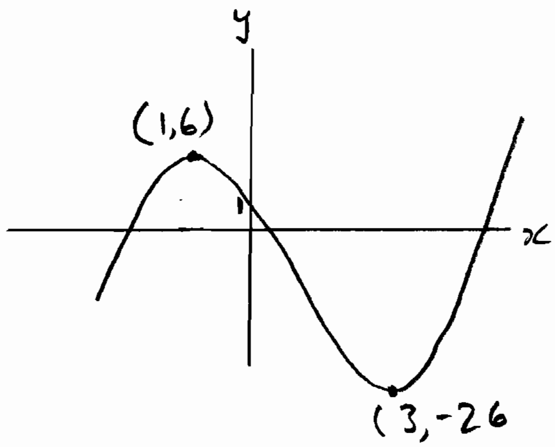
$$\Rightarrow \frac{d^2y}{dx^2} = 6x - 6$$

When $x = -1$, $\frac{d^2y}{dx^2} = 6(-1) - 6 = -12 < 0$

$\therefore$ a max at $(-1, 6)$

When $x = 3$, $\frac{d^2y}{dx^2} = 6(3) - 6 = +12 > 0$

$\therefore$ a min at $(3, -26)$



3. $y = x^2(x-4)^2$

$y = x^2(x^2 - 8x + 16)$

$y = x^4 - 8x^3 + 16x^2$

$\Rightarrow \frac{dy}{dx} = 4x^3 - 24x^2 + 32x$

At st.pt. $\frac{dy}{dx} = 0$

$\Rightarrow 4x^3 - 24x^2 + 32x = 0$

$\Rightarrow 4x(x^2 - 6x + 8) = 0$

$\Rightarrow 4x(x-2)(x-4) = 0$

$\Rightarrow x = 0, x = 2, x = 4$

when $x = 0$, $y = 0^2(0-4)^2 = 0$

when $x = 2$, $y = 2^2(2-4)^2 = 4 \times 4 = 16$

when $x = 4$ $y = 4^2(4-4)^2 = 0$

st pts $(0,0)$ $(2,16)$ $(4,0)$

$$\frac{dy}{dx} = 4x^3 - 24x^2 + 32x$$

$$\Rightarrow \frac{d^2y}{dx^2} = 12x^2 - 48x + 32$$

when $x = 0$, $\frac{d^2y}{dx^2} = +32 > 0 \therefore$ min

when $x = 2$, $\frac{d^2y}{dx^2} = 48 - 96 + 32 = -16 < 0 \therefore$ max

when $x = 4$ $\frac{d^2y}{dx^2} = 192 - 192 + 32 = 32 > 0 \therefore$ min

Min at $(0,0)$, Max at $(2,16)$, Min at $(4,0)$