GEOMETRIC PROGRESSIONS — EXERCISE

1. For the G.P. 7, 21, 63, 189, ...

   Find the 10^{th} term and the sum of the first 8 terms, S_8

2. For the GP 6, 3, 1·5, 0·75, ...

   Find the sum to infinity, S_∞

3. The 4th term of a GP is 96 and the 7th term is 6144. Find the sum of the first 8 terms, S_8.

4. The 1st term of a GP is 57 and the sum to infinity is 142.5. Find the common ratio.

5. The first term of a GP is 5 and the common ratio is 3. Which is the first term to exceed 250,000?

6. In the G.P 2, 6, 18, 54, ... how many terms are required for the sum to exceed 5,000,000 ?

GEOMETRIC PROGRESSIONS

EXERCISE

1.) G.P. 7, 21, 63, 189, ...

$a = 7$

$r = \frac{21}{7} = 3$

10^{th} term $= ar^9$

$= 7 \times 3^9$

$= 137,781$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_8 = \frac{7(3^8 - 1)}{3 - 1}$

$S_8 = 22,960$

2.) GP 6, 3, 1.5, 0.75, ...

$a = 6$

$r = \frac{3}{6} = \frac{1}{2}$

$S_\infty = \frac{a}{1-r}$

$= \frac{6}{1 - \frac{1}{2}}$

$= \frac{6}{\frac{1}{2}}$

$= 12$

3.) 4^{th} term $ar^3 = 96$ (1)

7^{th} term $ar^6 = 6144$ (2)

(2) ÷ (1) $\frac{ar^6}{ar^3}$ $\frac{6144}{96}$

$\Rightarrow r^3 = 64$

$\Rightarrow r = \sqrt[3]{64}$

$\Rightarrow r = 4$

Subst for r in (1)

$a \times 4^3 = 96$

$64a = 96$

$a = \frac{96}{64}$

$a = 1.5$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$S_8 = \frac{1.5(4^8 - 1)}{4 - 1}$

$S_8 = 32,767.5$

4.) $a = 57$, $S_\infty = 142.5$

$S_\infty = \frac{a}{1-r}$

$(1-r)S_\infty = a$

4 cont)

$$\Rightarrow 1 - r = \frac{a}{S_\infty}$$

$$\Rightarrow 1 - \frac{a}{S_\infty} = r$$

$$\Rightarrow 1 - \frac{57}{142.5} = r$$

$$\Rightarrow r = 0.6$$

5.) $a = 5, \quad r = 3$

Let n^{th} term be first to exceed 250,000

Then $ar^{n-1} > 250,000$

$$5 \times 3^{n-1} > 250,000$$

$$3^{n-1} > 50,000$$

Method 1 – trial and error

$$3^{10} = 59,049$$

$$3^{9} = 19,683$$

$$\therefore n - 1 = 10$$

$$n = 11$$

11^{th} term is first to exceed 250,000

Method 2 – using logarithms

$$3^{n-1} > 50,000$$

$$\Rightarrow \log_{10} 3^{n-1} > \log_{10} 50,000$$

$$\Rightarrow (n-1) \log_{10} 3 > \log_{10} 50,000$$

$$\Rightarrow n - 1 > \frac{\log_{10} 50,000}{\log_{10} 3}$$

$$\Rightarrow n - 1 > 9.85$$

$$\Rightarrow n > 10.85$$

$$\Rightarrow n = 11$$

since n is smallest whole number for which inequality is true.

$\therefore$ 11^{th} term is first to exceed 250,000

6.) $a = 2, \quad r = 3$

Let number of terms be n

$$\Rightarrow S_n > 5,000,000$$

$$\Rightarrow \frac{a(r^n - 1)}{r - 1} > 5,000,000$$

$$\Rightarrow \frac{2(3^n - 1)}{3 - 1} > 5,000,000$$

$$\Rightarrow 3^n - 1 > 5,000,000$$

6. cont) $\Rightarrow \quad 3^n > 5{,}000{,}001$

Method 1 – trial and error

$$3^{20} = 3{,}486{,}784{,}401$$
$$3^{15} = 14{,}348{,}907$$
$$3^{14} = 4{,}782{,}969$$

so $n = 15$

when 3^n first exceeds 5,000,001

$\therefore$ 15 terms are required

for sum to exceed 5,000,000

Method 2 – using logarithms

$$3^n > 5{,}000{,}001$$

$\Rightarrow \quad \log_{10} 3^n > \log 5{,}000{,}001$

$\Rightarrow \quad n \log_{10} 3 > \log 5{,}000{,}001$

$\Rightarrow \quad n > \dfrac{\log_{10} 5{,}000{,}001}{\log_{10} 3}$

$\Rightarrow \quad n > 14.04$

$\Rightarrow \quad n = 15$

first whole number satisfying inequality.

$\therefore$ 15 terms required