

# INTEGRATION BY PARTS

①

## Exercise

1.  $\int 4x \cos\left(\frac{x}{2}\right) dx$

2.  $\int 6x e^{-2x} dx$

3.  $\int 5x^4 \ln(2x) dx$

4.  $\int \frac{1}{x^2} \ln x dx$

5.  $\int_0^{\frac{\pi}{6}} x \sin 3x dx$

6.  $\int_1^2 3x e^{3x} dx$

7.  $\int_1^3 x^2 \ln x dx$

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Exercise Solutions

1.  $\int 4x \cos\left(\frac{x}{2}\right) dx$       Using  $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

Let  $u = 4x$       Let  $\frac{dv}{dx} = \cos\left(\frac{x}{2}\right)$

$\Rightarrow \frac{du}{dx} = 4$        $\Rightarrow v = 2 \sin\left(\frac{x}{2}\right)$

$$\begin{aligned} \int 4x \cos\left(\frac{x}{2}\right) dx &= 8x \sin\left(\frac{x}{2}\right) - \int 8 \sin\left(\frac{x}{2}\right) dx \\ &= 8x \sin\left(\frac{x}{2}\right) + 16 \cos\left(\frac{x}{2}\right) + c \end{aligned}$$

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2.  $\int 6x e^{-2x} dx$       Using  $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

Let  $u = 6x$       Let  $\frac{dv}{dx} = e^{-2x}$

$\Rightarrow \frac{du}{dx} = 6$        $\Rightarrow v = -\frac{1}{2} e^{-2x}$

$$\begin{aligned} \int 6x e^{-2x} dx &= -3x e^{-2x} + \int 3 e^{-2x} dx \\ &= -3x e^{-2x} - \frac{3}{2} e^{-2x} + c \end{aligned}$$

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3.  $\int 5x^4 \ln(2x) dx$  using  $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

Let  $u = \ln(2x)$

Let  $\frac{dv}{dx} = 5x^4$

$\Rightarrow \frac{du}{dx} = \frac{2}{2x} = \frac{1}{x}$

$\Rightarrow v = \frac{5x^5}{5} = x^5$

$$\begin{aligned} \int 5x^4 \ln(2x) dx &= x^5 \ln(2x) - \int x^5 \frac{1}{x} dx \\ &= x^5 \ln(2x) - \int x^4 dx \\ &= x^5 \ln(2x) - \frac{x^5}{5} + c \end{aligned}$$

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4.  $\int \frac{1}{x^2} \ln x dx$  using  $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

Let  $u = \ln x$

Let  $\frac{dv}{dx} = \frac{1}{x^2}$

$\Rightarrow \frac{du}{dx} = \frac{1}{x}$

$\Rightarrow v = -\frac{1}{x}$

$$\begin{aligned} \int \frac{1}{x^2} \ln x dx &= -\frac{1}{x} \ln x + \int \frac{1}{x} \times \frac{1}{x} dx \\ &= -\frac{1}{x} \ln x + \int \frac{1}{x^2} dx \\ &= -\frac{1}{x} \ln x - \frac{1}{x} + c \end{aligned}$$

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## INTEGRATION BY PARTS

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$$5. \int_0^{\frac{\pi}{6}} x \sin 3x \, dx$$

$$\text{using } \int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$$

$$\text{Let } u = x$$

$$\text{Let } \frac{dv}{dx} = \sin 3x$$

$$\Rightarrow \frac{du}{dx} = 1$$

$$\Rightarrow v = -\frac{1}{3} \cos 3x$$

$$\int_0^{\frac{\pi}{6}} x \sin 3x \, dx = \left[ -\frac{1}{3} x \cos 3x \right]_0^{\frac{\pi}{6}} + \int_0^{\frac{\pi}{6}} \frac{1}{3} \cos 3x \, dx$$

$$= \left[ -\frac{1}{3} x \cos 3x \right]_0^{\frac{\pi}{6}} + \left[ \frac{1}{9} \sin 3x \right]_0^{\frac{\pi}{6}}$$

$$= \left[ \left( -\frac{1}{3} \times \frac{\pi}{6} \cos \frac{\pi}{2} \right) - \left( -\frac{1}{3} \times 0 \times \cos 0 \right) \right] + \left[ \frac{1}{9} \sin \frac{\pi}{2} - \frac{1}{9} \sin 0 \right]$$

$$= \left[ 0 - 0 \right] + \left[ \frac{1}{9} - 0 \right]$$

$$= \frac{1}{9}$$

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INTEGRATION BY PARTS

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6.  $\int_1^2 3x e^{3x} dx$

using  $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

Let  $u = 3x$

Let  $\frac{dv}{dx} = e^{3x}$

$\Rightarrow \frac{du}{dx} = 3$

$\Rightarrow v = \frac{1}{3} e^{3x}$

$$\int_1^2 3x e^{3x} dx = \left[ x e^{3x} \right]_1^2 - \int_1^2 e^{3x} dx$$

$$= \left[ x e^{3x} \right]_1^2 - \left[ \frac{1}{3} e^{3x} \right]_1^2$$

$$= \left[ 2e^6 - e^3 \right] - \left[ \frac{1}{3} e^6 - \frac{1}{3} e^3 \right]$$

$$= \frac{6e^6}{3} - \frac{3e^3}{3} - \frac{1}{3} e^6 + \frac{1}{3} e^3$$

$$= \frac{5}{3} e^6 - \frac{2}{3} e^3$$

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## INTEGRATION BY PARTS

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7.  $\int_1^3 x^2 \ln x \, dx$  using  $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

Let  $u = \ln x$

Let  $\frac{dv}{dx} = x^2$

$\Rightarrow \frac{du}{dx} = \frac{1}{x}$

$\Rightarrow v = \frac{x^3}{3}$

$$\int_1^3 x^2 \ln x \, dx = \left[ \frac{x^3}{3} \ln x \right]_1^3 - \int_1^3 \frac{x^3}{3} \times \frac{1}{x} \, dx$$

$$= \left[ \frac{x^3}{3} \ln x \right]_1^3 - \int_1^3 \frac{x^2}{3} \, dx$$

$$= \left[ \frac{x^3}{3} \ln x \right]_1^3 - \left[ \frac{x^3}{9} \right]_1^3$$

$$= \left[ \frac{27}{3} \ln 3 - \frac{1}{3} \ln 1 \right] - \left[ \frac{27}{9} - \frac{1}{9} \right]$$

$$= 9 \ln 3 - 0 - \left[ \frac{26}{9} \right]$$

$$= 9 \ln 3 - \frac{26}{9}$$