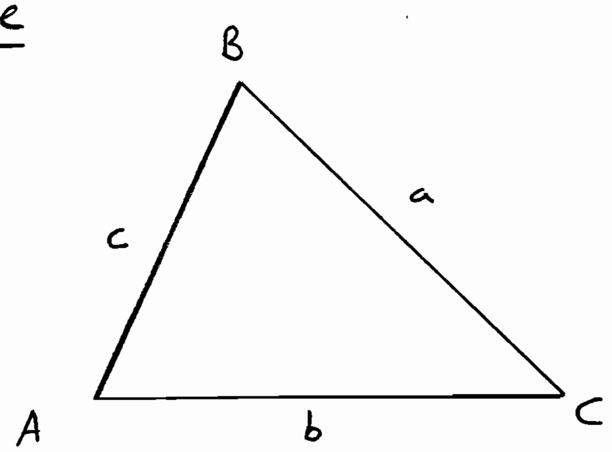
SINE AND COSINE RULES

TRANSCRIPT

Sine Rule

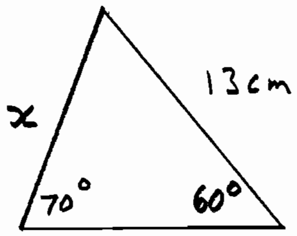


$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Valid for all triangles not just right-angled triangles!

Example 1

Finding a side



Find x

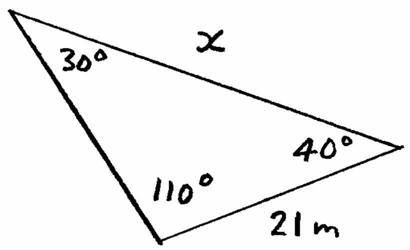
Sine Rule $\quad \dfrac{13}{\sin 70^\circ} = \dfrac{x}{\sin 60^\circ}$

$$\Rightarrow \quad \frac{13}{\sin 70^\circ} \times \sin 60^\circ = x$$

$$\Rightarrow \quad x = 11.98 \text{ cm}$$

Example 2

Finding a side



Find x

Sine Rule

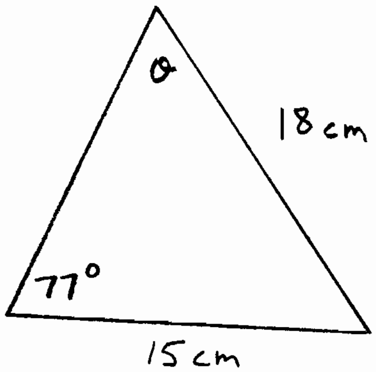
$$\frac{21}{\sin 30^\circ} = \frac{x}{\sin 110^\circ}$$

$$\Rightarrow \frac{21}{\sin 30^\circ} \times \sin 110^\circ = x$$

$$\Rightarrow x = 39.47 \text{ m}$$

Example 3

Finding an angle



Find angle θ

Sine Rule

$$\frac{15}{\sin\theta} = \frac{18}{\sin 77^\circ}$$

$$\Rightarrow 15\sin 77^\circ = 18\sin\theta$$

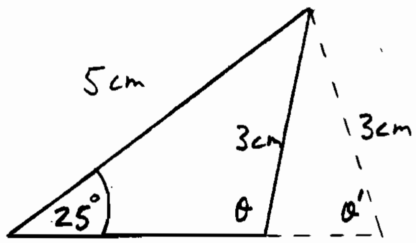
$$\Rightarrow \frac{15\sin 77^\circ}{18} = \sin\theta$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{15\sin 77^\circ}{18}\right)$$

$$\Rightarrow \theta = 54.3^\circ$$

Example 4 — Finding an angle

This example is called the ambiguous case of sine rule



Two different triangles can be constructed with a $25°$ angle adjacent to a 5cm side and opposite a 3cm side

In this case we cannot be sure whether the angle opposite the 5cm side is the obtuse angle θ or the acute angle θ'

Sine Rule

$$\frac{3}{\sin 25°} = \frac{5}{\sin\theta}$$

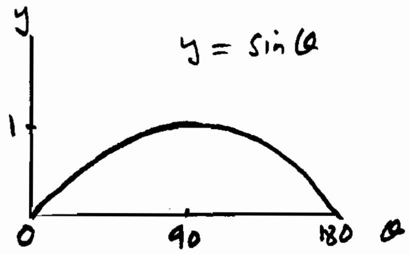
$$\Rightarrow \quad 3\sin\theta = 5\sin 25°$$

$$\Rightarrow \quad \sin\theta = \frac{5\sin 25°}{3}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{5\sin 25°}{3}\right)$$

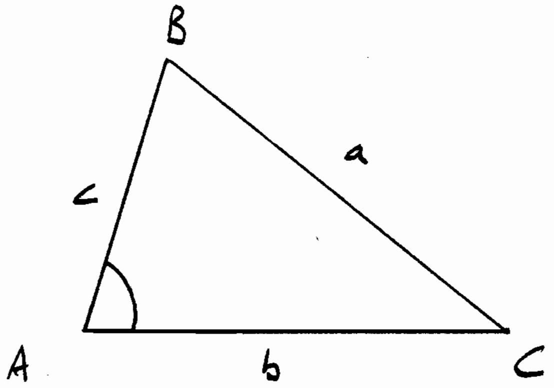
$$\Rightarrow \quad \theta = 44.8° \quad \text{or} \quad 180° - 44.8° = 135.2°$$

This is because $\sin\theta = \sin(180 - \theta)$



To check whether the obtuse angle is possible, we can add $135.2°$ to $25° = 160.2°$ to ensure the $180°$ angle sum for the triangle has not been exceeded.

Cosine Rule



$$a^2 = b^2 + c^2 - 2bc\cos A$$

This formula enables us to calculate a side of any triangle provided that we know the other two sides and the angle between those known sides.

The formula can be rearranged to find the angle form of the cosine rule.

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Rightarrow \quad 2bc\cos A = b^2 + c^2 - a^2$$

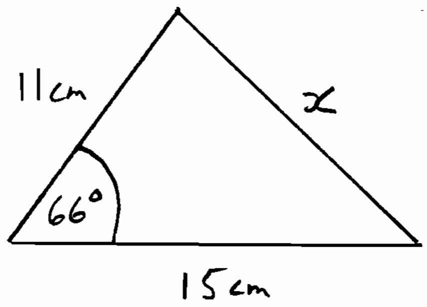
$$\Rightarrow \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Angle form of cosine rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Example 5

Finding a side



Find x

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

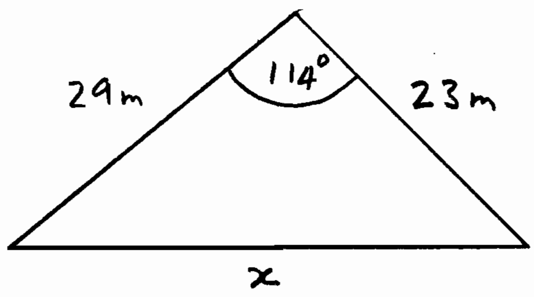
$x^2 = 11^2 + 15^2 - 2 \times 11 \times 15 \times \cos 66^\circ$

$x^2 = 211.78$

$x = 14.55$ cm

Example 6

Finding a side



Find x

Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$

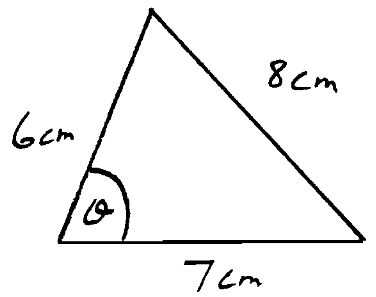
$x^2 = 29^2 + 23^2 - 2 \times 29 \times 23 \times \cos 114^\circ$

$x^2 = 1912.59$

$x = 43.73$ m

Example 7

Finding an angle



Find θ

Cosine Rule

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos\theta = \frac{6^2+7^2-8^2}{2\times 6\times 7}$$

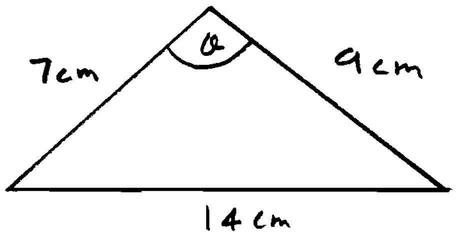
$$\cos\theta = 0.25$$

$$\theta = \cos^{-1} 0.25$$

$$\theta = 75.5^\circ$$

Example 8

Finding an angle



Find θ

Cosine Rule

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos\theta = \frac{7^2+9^2-14^2}{2\times 7\times 9}$$

$$\cos\theta = -0.5238$$

$$\theta = \cos^{-1}(-0.5238)$$

$$\theta = 121.6^\circ$$
